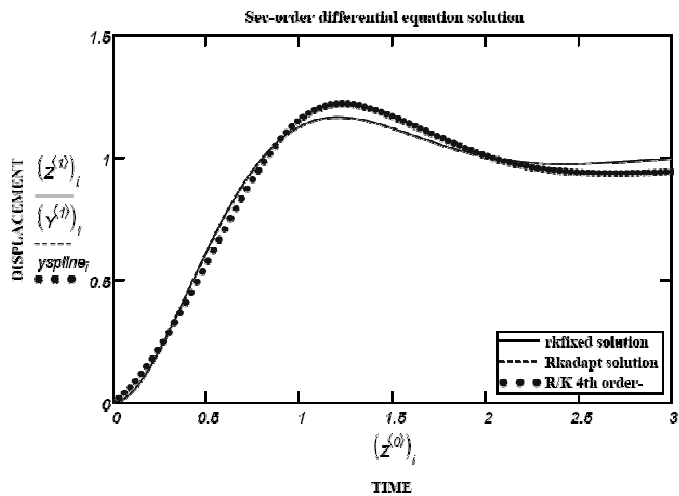
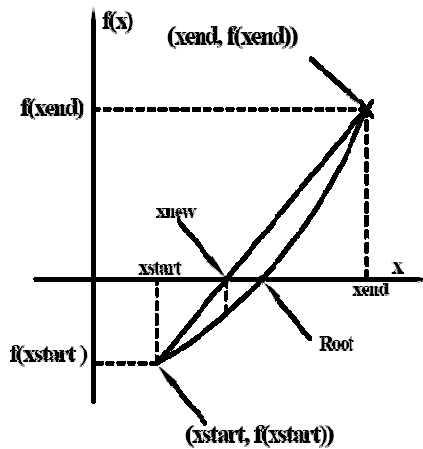


AN INTRODUCTION TO NUMERICAL METHODS USING MATHCAD

Mathcad Release 14

$h := 1..5$ Various orders of approximation generated by Taylor series
 $x := 0, 1..5$ approach versus true values of given function

	zeroth order	first order	second order	third order	fourth order	fifth order	True Value
$x =$	$ftaylor0(x)$	$ftaylor1(x)$	$ftaylor2(x)$	$ftaylor3(x)$	$ftaylor4(x)$	$ftaylor5(x)$	$f(x) =$
$h =$	0	1.25	1.25	1.25	1.25	1.25	1.25
1	1	1.25	1	0.75	0.58	0.43	0.43
2	2	1.25	0.75	-0.25	-1.61	-4.01	-4.01
3	3	1.25	0.5	-1.75	-6.34	-18.49	-18.49
4	4	1.25	0.25	-3.75	-14.63	-53.03	-53.03
5	5	1.25	0	-6.25	-27.5	-121.25	-121.25



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