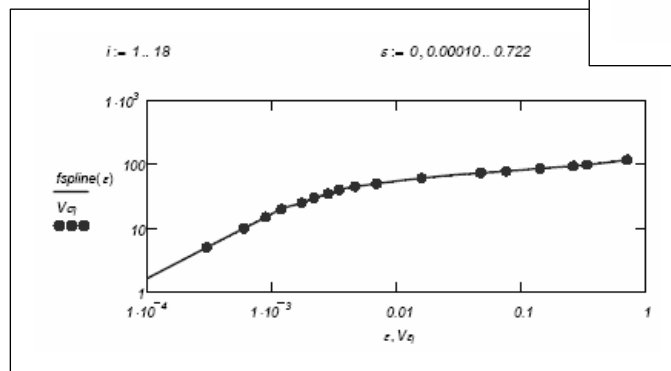
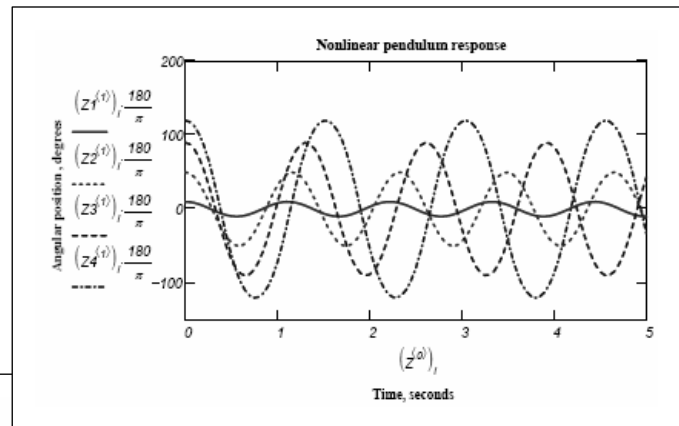
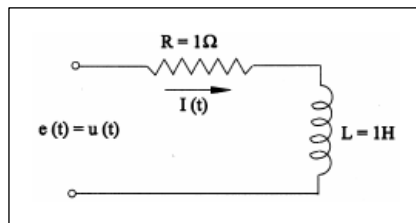


AN INTRODUCTION TO NUMERICAL METHODS USING MATHCAD

Mathcad Release 13



$$\begin{bmatrix} n & \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i & \sum_{i=1}^n (x_i)^2 \end{bmatrix} \begin{pmatrix} C \\ B \end{pmatrix} = \begin{bmatrix} \sum_{i=1}^n (y_i) \\ \sum_{i=1}^n [(y_i) \cdot x_i] \end{bmatrix} \quad (n=6)$$

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